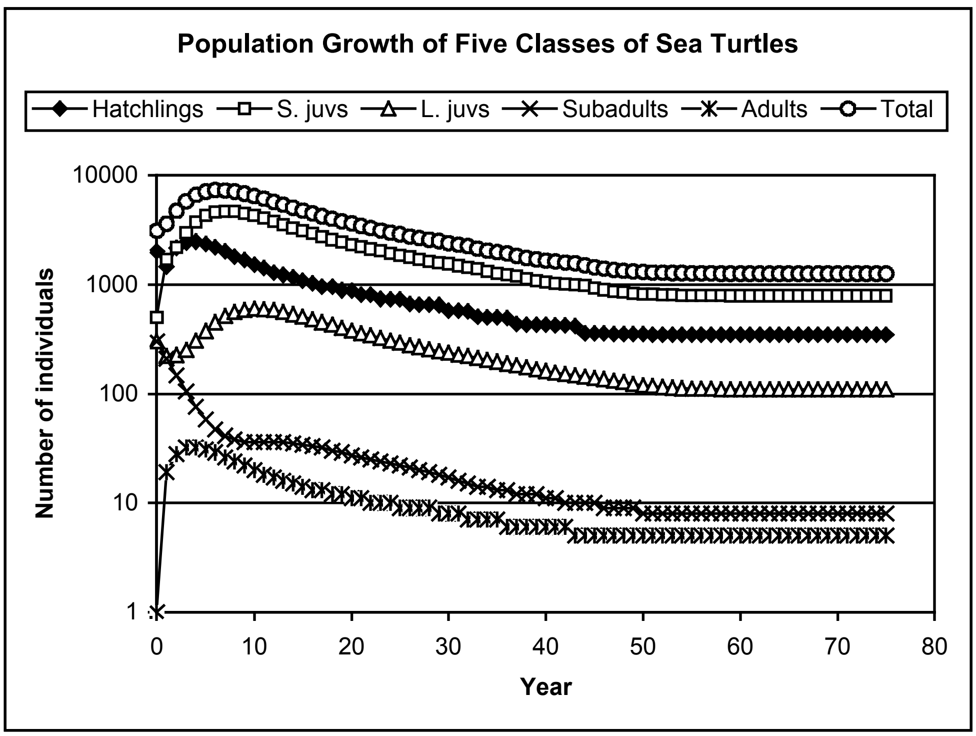
Answers to Exercise 14

Stage-Structured Matrix Models

1. The assumptions of the matrix model are the same as those for exponential growth, except that age structure has been accounted for. In other words, once we account for age, the population's finite rate of increase can be calculated, and the population will increase geometrically, decrease geometrically, or remain stable over time. As with the exponential growth model, the matrix model assumes that resources are unlimited. For this model, we have also assumed that individuals give birth the moment they enter a new age class and that population censuses occur immediately after birth.

2. At year 60, the population's finite rate of increase, λ, is 1. This stabilized growth is readily apparent by examining the semi-log graph, where the projection lines for each stage class become parallel. The horizontal nature of the lines indicates that the population is neither increasing nor decreasing over time; it remains constant. At stable distribution, the population is dominated by small juveniles, then hatchlings, large juveniles, subadults, and adults. For all of these age classes, $\lambda = 1$.

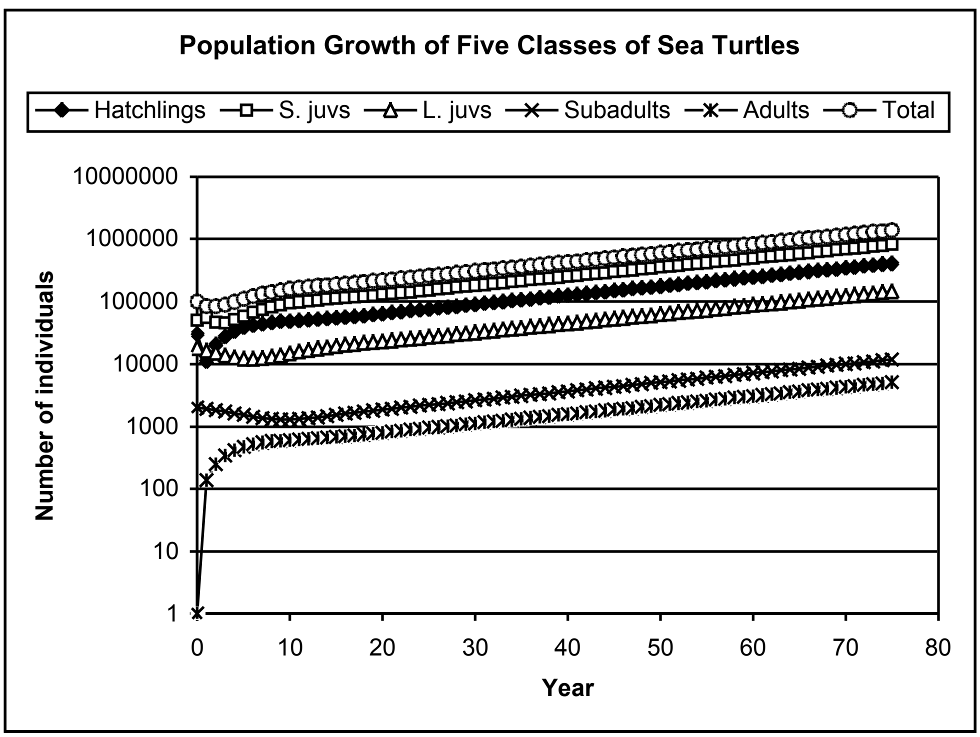


3. When the population reaches a stable distribution, it consists of 27.6% hatchlings, 62.7% small juveniles, 8.7% large juveniles, 0.64% subadults, and 0.397% adults. This vector of abundances is called a *right eigenvector* for the **L** matrix.

4. You should see that the initial population vector has no influence on the stable stage distribution, or on λ once the stable age distribution has been attained. However, the initial dynamics (λ_t, pre-stable distribution) are strongly affected by the initial population vector. With an initial distribution of 75 hatchlings, 1 small juvenile, 1 large juvenile, 1 subadult, and 1 adult, λ in year 1 is 1.53, or a 53% increase over time. (Recall that the value for year 1 had previously been 1.16.).

   Early irregularities in the stage structure and growth rate are instabilities reflecting initial departure from a stable stage distribution. The irregularities are not due to stochasticity in fertility or survival rates, because these rates remained fixed in the Lefkovitch matrix. How far the stage distribution differs from a stable distribution has important population management implications because, initially, the composition of individuals affects whether the population will increase, decrease, or remain stable, given the set of parameters entered into the Lefkovitch matrix.

5. The population reaches a stable distribution at approximately year 13. At t his time it is composed of approximately 29.1% hatchlings, 59.3% small juveniles, 10.4% large juveniles, 0.85% subadults, and 0.37% adults. Each of these stages increases by 3% per year, and their proportions remain constant over time. Thus, TEDS have increased the growth rate by 3% per year.



6. In order to increase λ to 1.03, and keeping the original *P* values in the **L** matrix, fertilities of subadults and adults would need to increased by a factor of around 3.5. We obtained this number by setting up spreadsheet headings as shown below. Cell G4 is a factor to be multiplied by the original fertility value. Cell E4 has the formula **=4.665*G4**, and cell F4 has the formula **=61.896*G4**. Enter values in cell G4 until the stabilized λ = 1.03. (You can also use Excel's Solver function to determine this answer.)

	B	C	D	E	F	G
3	***F*(*h*)**	***F*(*sj*)**	***F*(*lj*)**	***F*(*sa*)**	***F*(*a*)**	**Factor**
4	0	0	0	16.3275	216.64	3.5
5	0.675	0.703	0	0	0	
6	0	0.047	0.657	0	0	
7	0	0	0.019	0.682	0	
8	0	0	0	0.061	0.8091	

7. N/A