Answers to Exercise 13

Age-Structured Matrix Models

1. All age classes are increasing geometrically over time. Although λ is difficult to determine by examining the graph, it must be greater than 1 in order for geometric increases to occur.

2. At year 19 the population's finite rate of increase stabilizes at 1.180768. This is the asymptotic growth rate. Signs of stabilization appear as early as year 7. This stabilized growth is readily apparent by examining the semi-log graph, where the projection lines for each age class become parallel. At that point, individuals in age class 1, followed by age classes 2, 3, and 4, dominate the population.

3. The stable age distribution is

	H	I	J	K
10	**Stable Age Distribution**			
11	**1**	**2**	**3**	**4**
12	0.493786	0.33455	0.14166719	0.0299947

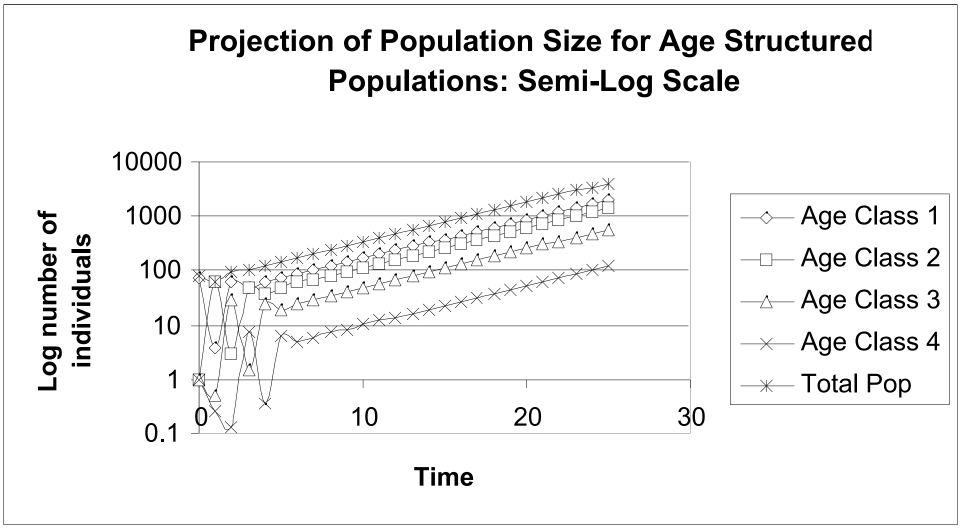
In other words, when the population has reached a stable distribution, 49% of the population consists of individuals from age class 1, 33% of the population consists of individuals from age class 2, 14% of the population consists of individuals from age class 3, and 3% of the population consists of individuals from age class 4. Note that these values sum to 100%. This is called the stable age vector. The formulae used in these calculations included

•H13 =**B37/$F$37** •J13 =**D37/$F$37**
•I13 =**C37/$F$37** •K13 =**E37/$F$37**

4. You should see that the initial population vector has no influence on the stable age distribution or λ once the stable age distribution has been attained. However, the initial dynamics (before the distribution stabilizes) are strongly affected by the initial population vector.

With an initial distribution of 75, 1, 1, and 1 individuals in age classes 1 through 4, respectively, λ in year 1 is 0.827; the population decreased by approximately 17%. (Recall that with the initial distribution of 45 individuals in age class 1, 18 individuals in age class 2, 11 individuals in age class 3, and 4 individuals in age class 4, λ in year 1 was 1.116.) Early irregularities in the age structure and growth rate are instabilities reflecting initial departure from a stable age distribution; the irregularities are *not* due to stochasticity in fertility or survival rates because these rates remained fixed in the Leslie matrix.

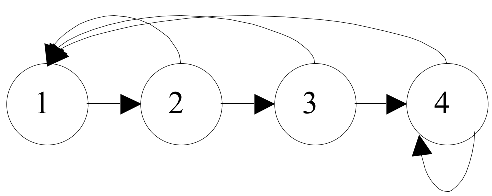
How far the age distribution is from a stable distribution has important implications for population management because, initially, the composition of individuals affects whether the population will increase, decrease, or remain stable, given the set of parameters entered into the Leslie matrix.



Graph for Question 4

5. The assumptions of the matrix model are the same as those for exponential growth, except that we have accounted for age structure. In other words, once we account for age, the population's finite rate of increase can be calculated, and the population will increase geometrically, decrease geometrically, or remain stable over time. As with the exponential growth model, this matrix model assumes that resources are unlimited. For this model, we have also assumed that individuals give birth the moment they enter a new age class, and that population censuses occur immediately after birth. We have also assumed that, within an age class, birth and death rates are homogeneous.

6. The life cycle diagram would be adjusted as follows:



The adjusted Leslie matrix has the form

	A	B	C	D	E
3		**Age class**			
4		**1**	**2**	**3**	**4**
5		0	1	1.5	1.2
6	**A** =	0.8	0	0	0
7		0	0.5	0	0
8		0	0	0.25	0.25

For this matrix, λ stabilizes at 1.188 compared to 1.180. The stable age distribution has the following proportions:

	H	I	J	K
11	**1**	**2**	**3**	**4**
12	0.49218	0.33131	0.13938385	0.03713087

Thus, there are slightly more individuals in age class 4 (3.71%) than previously (2.99%), and slightly fewer individuals in age classes 1, 2, and 3 than previously.