

## Answers to Exercise 10

### *Predator-Prey Dynamics*

#### *Answers to Base Questions (Questions 1–6, p. 140)*

1. Increase and decrease the value of  $R$  by small increments and observe the changes in your graphs.

You should find that smaller values of  $R$  delay the extinction of both populations. This happens because lower  $R$ -values slow not only the growth of the prey population but also that of the predator population. Review Equation 2; note that the per capita growth rate of the predator population ( $afV_t$ ) is a function of prey population size ( $V_t$ ). Thus, a slowly growing prey population implies a slowly growing predator population, and the whole system changes more slowly.

Larger  $R$  values bring about extinction ever more quickly. Faster prey population growth allows faster predator population growth, and the more numerous predators kill off the prey all the faster, leading to their own demise. Faster prey population growth decreases the stability of the system.

This counterintuitive result comes from the positive feedback between the two populations. An increase in prey leads to an increase in predators, which leads to more prey killed. Also, both populations have a built-in time lag in the discrete-time model, so that reproduction in each depends on the size of the population one time unit earlier. These two factors produce a model in which both populations go extinct sooner or later.

In summary, the predators kill off all the prey and then starve to death.

2. Increase and decrease the value of  $q$  by small increments and observe the changes in your graphs. You should observe that smaller values of  $q$  delay extinction, and larger values lead to more rapid extinction of both prey and predator.

Another counterintuitive result! The explanation is that if predators starve more rapidly during times of low prey abundance, this allows the prey population to recover to larger values before the predator population can respond. Once the prey population is very large, the predator population explodes, and they eat all the prey and then starve quickly.

3. Increase and decrease the value of  $f$  by small increments and observe the changes in your graphs.

Smaller values of  $f$  delay, and larger values hasten, extinction of both populations. This result, at least, makes sense. If predators are more efficient at converting food into offspring, then their population will grow faster. A larger population of predators

will kill more prey, leading to the inevitable result.

4. Increase and decrease the value of  $a$  by small increments and observe the changes in your graphs.

Paradoxically, if the predator gets much better or much worse at finding prey (higher or lower values of  $a$ ), both species go extinct faster.

For higher values of  $a$ , this results from the predators finding and killing prey more quickly. For lower values of  $a$ , the prey population explodes, but then so does the predator population. At some point, there are so many predators that even with poor hunting abilities they wipe out the prey.

Whenever the prey go extinct, the predators are not far behind.

5. Try different values of  $C_0$  and  $V_0$ . If you start with populations at the point where the ZNGIs cross, they will remain there, at equilibrium. As the initial populations move farther from the crossing point, their cycles become more exaggerated.

The model is more sensitive to changes in  $C_0$  than to changes in  $V_0$ .

6. The ultimate outcome in the discrete-time model is the extinction of both species. Even in cases where extinction does not occur within the 100 time units modeled, you can see that it will occur eventually, because the amplitudes of the population cycles increase with time. Therefore, they must eventually go to zero.

On the graph of  $C_t$  versus  $V_t$ , the trajectory spirals outward, away from the unstable equilibrium point where the ZNGIs cross. Eventually, the trajectory must intersect an axis. This is in sharp contrast to the continuous-time model, in which the amplitudes of the population cycles are constant and the  $C_t$  versus  $V_t$  trajectories are closed ellipses.

### ***Model with Prey Refuges (p. 141)***

7. You should find that refuges prevent the prey from going extinct, and thus also prevent the predators from going extinct. In this sense, at least, the new model is more stable than the original.

However, refuges do nothing to lessen the extreme swings in population sizes of either species. Predators reduce the prey population to the minimum specified in your prey formula, and then the predator population declines exponentially until it is small enough that the prey population can increase. The prey population then grows exponentially, as does the predator population, after a delay. The predators then reduce the prey population to its minimum again.

This cycle repeats indefinitely. It may be easier to see the exponential increases and decreases if you change the vertical axes of your population versus time graph to

logarithmic scales. Exponential changes will appear as straight lines.

***Model with Prey Carrying Capacity (p. 144)***

8. The addition of a prey carrying capacity radically changes the behavior of the model. Now, instead of the amplitudes of the population cycles increasing over time, they decrease.

On the graph of  $C_t$  versus  $V_t$ , the trajectory spirals inward toward a stable equilibrium at the point where the ZNGIs cross. However, the populations may still go extinct if their cycles start out large enough.

***Model with Prey and Predator Carrying Capacities (p. 148)***

9. This model produce the most stable system one yet. Populations cycles are damped more quickly (the trajectory spirals inward more quickly) to equilibrium. Extinctions are still possible, for example if attack or starvation rates are high.
10. All the models predict that predator-prey systems should be highly volatile, displaying either population cycles or extinctions, unless one or both species are limited by other factors.

A potentially stabilizing factor not included in these models is the presence of more than one prey species. If the predator switches to a more abundant prey species when one prey species becomes rare, would that prevent extinctions and promote stability? Could you build this idea into this model?