

29

HARVEST MODELS

Objectives

- Understand the concept of recruitment rate and its relationship to sustainable harvest.
- Understand the concepts of maximum sustainable yield, fixed-quota harvest, and fixed-effort harvest.
- Confirm model predictions of maximum sustainable yield in both harvesting models.
- Explore the sensitivity of these models to errors in estimates of population parameters, or to year-to-year variation in these parameters.
- Understand how model sensitivities affect the risk of extinction of the harvested population.

Suggested Preliminary Exercise: Logistic Population Models

INTRODUCTION

In this exercise, you will set up spreadsheet models of populations that are being harvested or hunted by humans. Fishery and wildlife managers use models similar to these, although more sophisticated, to manage populations of commercially exploited fishes such as salmon, as well as populations of deer, elk, and other game animals.

Managers face two contradictory demands: to maximize the harvest for current users, and to sustain the harvested population for future users. To balance these demands, they try to estimate the **maximum sustainable yield (MSY)**—the largest harvest that users can take indefinitely without causing extinction of the harvested population. They begin with the simple form of the logistic model (shown here in the discrete-time form),

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) \quad \text{Equation 1}$$

and add a term representing either a **fixed-quota harvest** or a **fixed-effort harvest**.

In a fixed-quota harvest, managers allow users to take the same total number of individuals from the population each year, by setting catch limits or bag limits. Mathematically, fixed-quota harvesting looks like this:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - Q \quad \text{Equation 2}$$

where Q is a constant parameter representing the number of individuals that users are allowed to remove, or harvest, from the population. Managers regulate the population by adjusting the quota Q in the range of zero to K .

In a fixed-effort harvest, managers regulate the effort that users can devote to harvesting. In fisheries, managers often regulate the number of boats that can fish in a certain area, what kinds of gear they can use, and how many days they can fish. In game management, agencies issue limited numbers of hunting licenses and limit the number of days of the hunting season. Mathematically, fixed-effort harvesting looks like this:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - EN_t \quad \text{Equation 3}$$

where E is a constant parameter (≤ 1) representing the total effort expended by harvesters. In other words, we assume that by expending a certain effort, users will end up harvesting some fraction of the total population at the time. Managers then, in effect, regulate what fraction users take by adjusting E in the range of zero to one.

PROCEDURES

Your job in this exercise is to set up models of fixed-quota and fixed-effort harvesting and use them to answer some questions:

- What is the maximum sustainable yield? Does it differ between the two models?
- How will the harvested population respond to the removal of the maximum sustainable yield using a fixed-quota harvest? Using a fixed-effort harvest? Do the answers to these questions differ, or do the two models make the same prediction?
- What happens to the population in each model if managers overestimate the population's growth rate or carrying capacity? Are fixed-quota and fixed-effort harvest models equally sensitive to such errors, or is one more robust than the other?
- Can managers build a safety margin into these models to protect against errors in estimating population growth rate or carrying capacity or to protect against year-to-year variation in these parameters? How effective are such safety margins in each model?
- What happens to the population in each model if environmental conditions vary from year to year? Are the models equally sensitive to such variation, or is one more robust?

This exercise is set up somewhat differently from others in this book in that working through questions and their answers is intrinsic to the process here. You will be asked to answer specific questions based on the data in your models. You should try to find the answers for yourself, but detailed explanations of the answers are provided at the end of the exercise. Your answers to the specific questions should allow you to provide concrete answers to the larger questions posed above.

In the first part of the exercise, you will set up spreadsheet models of fixed-quota and fixed-effort harvesting and explore the behavior of each model to get an intuitive feel for how the models operate. We will then verify your intuition analytically.

In the final section of the exercise, you will add annual variation in population growth and carrying capacity to the model to see what effects that has on sustainability of harvest.

As always, save your work frequently to disk.

INSTRUCTIONS

A. Set up the harvest model parameters.

1. Open a new spreadsheet and set up titles and column headings as shown in Figure 1.

2. Set up a linear series from 0 to 50 in column A (cells A12–A62).

3. Enter the values shown in Figure 1 for the parameters R , K , Q , and E .

4. Enter the initial population size (N_0) of 1.00 into cell B12.

5. In cells B13–B62, enter a formula to calculate the population size at time 1 (N_1).

6. In cells C12–C61, enter a formula for ΔN .

ANNOTATION

These are all literals, so just select the appropriate cells and type them in.

	A	B	C	D	E	F	G	H	I
1	Harvest Models								
2	These models assume a logistically growing population that is harvested in one of two ways:								
3	1) Fixed-quota harvest								
4	2) Fixed-effort harvest								
5									
6	$R =$	0.5							
7	$K =$	2000							
8	Unharvested			Harvested					
9			Fixed-quota	$Q =$	250	Fixed-effort	$E =$	0.25	
10					Actual			Actual	
11	Time	N_t	ΔN_t	N_t	Quota	harvest	N_t	Effort line	harvest
12	0	1.00							
13	1								
14	2								

Figure 1

In cell A12 enter the value zero.

In cell A13 enter the formula =A12+1. Copy the formula in cell A13 into cells A14–A62.

Enter the values shown in Figure 1 into cells B6, B7, F9, and I9.

Column B will model growth of an **unharvested** population with the given values of R and K . Use this population for comparison to the harvested populations.

In cell B13, enter the formula =B12+\$B\$6*B12*(\$B\$7-B12)/\$B\$7. Copy the formula into cells B14–B62.

This formula corresponds to Equation 1:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right)$$

This is the *unharvested* population, so your formula does not include Q or E .

In cell C12, enter the formula =B13-B12. Copy the formula into cells C13–C61.

Column C will model ΔN for an unharvested population. In this context, ΔN is called the **recruitment rate** because it represents the net number of new members that are added, or recruited, into the population in the current time interval. The recruitment rate is important because it is also the number of individuals that can be taken from the population without reducing its size. In other words, if we take from a population exactly as many individuals as are added by birth, less the number that die for reasons unrelated to our harvest, then the size of the population will remain unchanged.

7. In cell D12, enter a formula to start the fixed-quota harvested population at its carrying capacity.

8. In cells E12–E62, enter a formula to echo the fixed quota in cell F9.

9. In cell G12, enter a formula to start the fixed-effort harvested population at its carrying capacity.

10. In cell H12, enter a formula to calculate the number of individuals that fixed-effort harvesters would be allowed to take from the unharvested population in column B.

11. Copy the formula in cell H12 down the column.

B. Calculate the sizes of harvested populations.

1. In cell D13, enter a formula for the fixed-quota harvested population at time 1.

In cell D12, enter the formula **=B\$7**.

This will force this harvested population to begin at its carrying capacity. It is important to use a formula rather than typing in the value so that when you change K later, the spreadsheet will automatically change all the populations in concert.

Managers often assume for convenience that a previously unharvested population is at its carrying capacity. This may not be the case, and later in this exercise you will see why it can be a risky assumption.

In cell E12, enter the formula **=F\$9**. Copy this formula into cells E13 through E62.

This represents the allowed harvest under a fixed-quota scheme. Again you use a formula rather than typing in a value so that when you change Q later, the spreadsheet will automatically change the entire column.

As implied by the name “fixed-quota,” the harvest quota does not change over time or in response to population size. You will use this column to compare the allowable harvest to the recruitment rate.

In cell G12 enter the formula **=B\$7**.

This will force this harvested population to begin at its carrying capacity, also.

In cell H12, enter the formula **=I\$9*B12**.

This calculates the allowed harvest (EN_t) under a fixed-effort scheme. As discussed in the Introduction, a fixed-effort harvest is a fixed proportion of the population size. The parameter E , in cell I9, represents that proportion.

This column shows the allowable harvest for any population size from zero to K , so it uses column B rather than column G. This is called the fixed-effort line. You will use this column to compare the allowable harvest to the recruitment rate.

If you were to simply enter the spreadsheet equivalents of Equations 2 and 3, these numbers would have the potential to become less than zero. This makes no sense biologically, and can produce some strange graphs. Therefore, we want to restrict these calculations to values greater than or equal to zero. We do this in the following steps by using the spreadsheet function **IF0**.

You should read through this entire annotation carefully before actually entering the formula. Lightface type in the formula is used for explanatory purposes.

In cell D13, enter the formula **=IF(D12+B\$6*D12*(B\$7-D12)/B\$7-F\$9>0,D12+B\$6*D12*(B\$7-D12)/B\$7-F\$9,0)**.

This looks horrible, but let’s pull it apart. The first portion in boldface type—immediately after the first parenthesis and before “>0”—corresponds to Equation 2:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - Q$$

This is the formula for logistic growth minus the fixed quota. Check the cell addresses and confirm this for yourself. The second boldface section, between the commas, is the same thing. By adding the zeros (lightface type), the entire formula says, “Calculate what the harvested population would be, and if that number is greater than zero, use

2. Copy the formula from cell D13 into cells D14–D62.

3. In cell G13 enter a formula for the fixed-effort harvested population at time 1. (Again, you will use the **IF()** function to ensure that the population never goes below zero.)

4. Copy the formula from cell G13 into cells G14–G62.

5. In cells F12–F62, enter a formula for the actual catch from the fixed-quota harvested population.

6. In cells I12–I62, enter a formula for the actual catch from the fixed-effort harvested population.

7. Your spreadsheet is complete. **Be sure to save your work now!**

C. Create graphs.

1. Make an XY graph of population size and actual catch against time for the fixed-quota population.

it. If it's less than or equal to zero, use 0 instead." This is how you will prevent your harvested populations from going negative.

To enter this formula, you could just type it all in, but there's a slightly easier way. Begin typing the formula as shown, up through the >0. Type a comma, and then in the formula bar, click and drag the cursor over the formula for the population size (leave out the >0). Copy the formula, and then paste it after the comma. Then type the second comma, followed by a zero. Close the parentheses and you're finished. This is not only quicker, but will save you from many typing errors.

In cell G13, enter the formula **=IF(G12+\$B\$6*G12*(B\$7-G12)/B\$7-I12>0,G12+\$B\$6*G12*(B\$7-G12)/B\$7-I12,0)**.

The boldface portions of this formula, **G12+\$B\$6*G12*(B\$7-G12)/B\$7-I12**, correspond to Equation 3:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - EN_t$$

Note that the item immediately after B\$7 above is a cell address, I12, not the number, 112. As in the fixed-quota formula, we use the **IF()** function to prevent the harvested population from going negative. The entire formula says, "Calculate what the harvested population would be, and if that's greater than zero, use it. If it's less than or equal to zero, use 0 instead." You can use the shortcut given in Step 1 to enter this formula.

In cell F12, enter the formula **=IF(D12>\$F\$9,\$F\$9,D12)**. Copy the formula into cells F13–F62.

The actual catch will be the quota, unless the population is smaller than that, in which case the catch will be the entire population, so again we use the **IF()** function. The rationale for this seemingly draconian harvest is that if harvesters did not know that the population had fallen below the allowed harvest, they would try to harvest the quota and end up harvesting the entire population.

In cell I12, enter the formula **=IF(G12>\$I\$9*\$G12,\$I\$9*G12,G12)**. Copy the formula into cells I13–I62.

Another **IF()** formula: The actual catch will be the product of the effort times the population size, unless population is smaller than that, in which case the catch will be the entire population. The rationale is the same as above.

Select cells A11 through A62.

While holding down the **⌘** key (Macintosh) or the control key (Windows), select cells D11 through D62.

While holding down the **⌘** key (Macintosh) or the control key (Windows), select cells F11 through F62.

Follow the usual procedure for making and editing an XY graph (scatterplot). Your graph should resemble Figure 2.

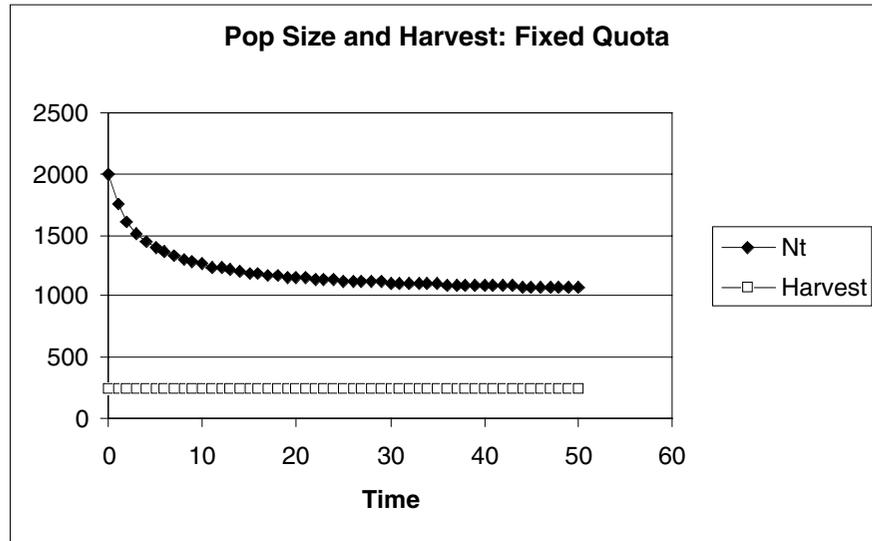


Figure 2

2. Make an XY graph of population size and actual catch against time for the fixed-effort population.

Select cells A11 through A62.

While holding down the key (Macintosh) or the control key (Windows), select cells G11 through G62.

While holding down the key (Macintosh) or the control key (Windows), select cells I11 through I62.

Follow the usual procedure for making and editing an XY graph (scatterplot). Your graph should resemble Figure 3.

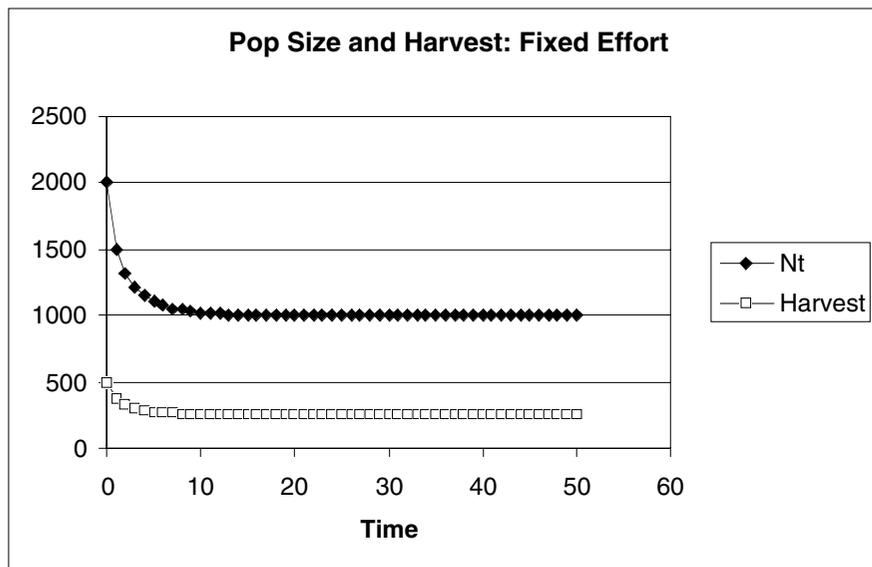


Figure 3

3. Make an XY graph of recruitment rate versus population size for the

Select cells B11 through C61.

While holding down the key (Macintosh) or the control key (Windows), select cells E11 through E61.

unharvested population and plot the fixed quota on the same graph.

Follow the usual procedure for making and editing an XY graph (scatterplot). Your graph should resemble Figure 4.

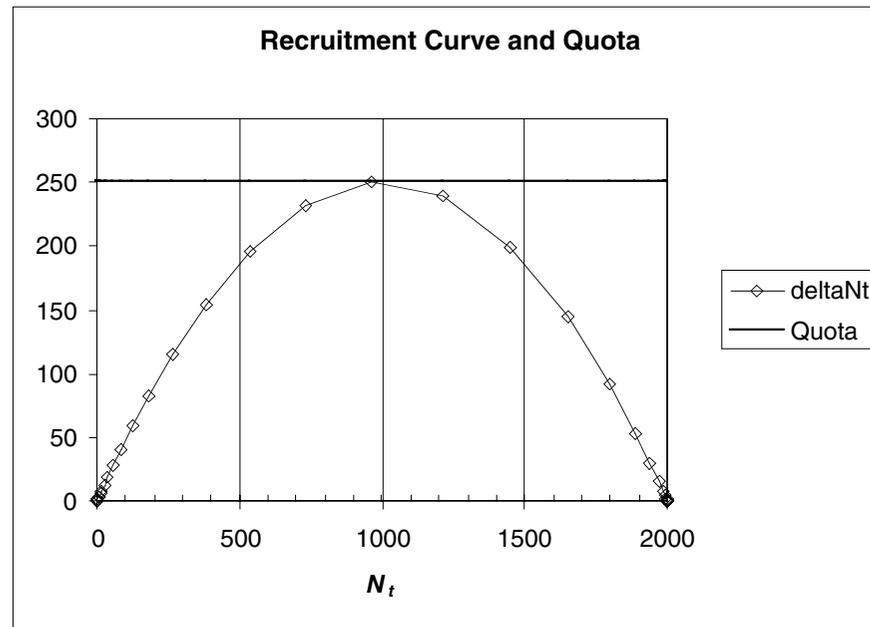


Figure 4 Note that the quota line (horizontal line) just touches the recruitment curve at its peak. This observation will be important in interpreting later developments of the model.

4. Make a separate XY graph of recruitment rate versus population size for the *unharvested* population, and plot the fixed-effort line on the same graph.

Select cells B11 through C61.

While holding down the **⌘** key (Macintosh) or the control key (Windows), select cells E11 through E61.

Follow the usual procedure for making and editing an XY graph (scatterplot). Your graph should resemble Figure 5.

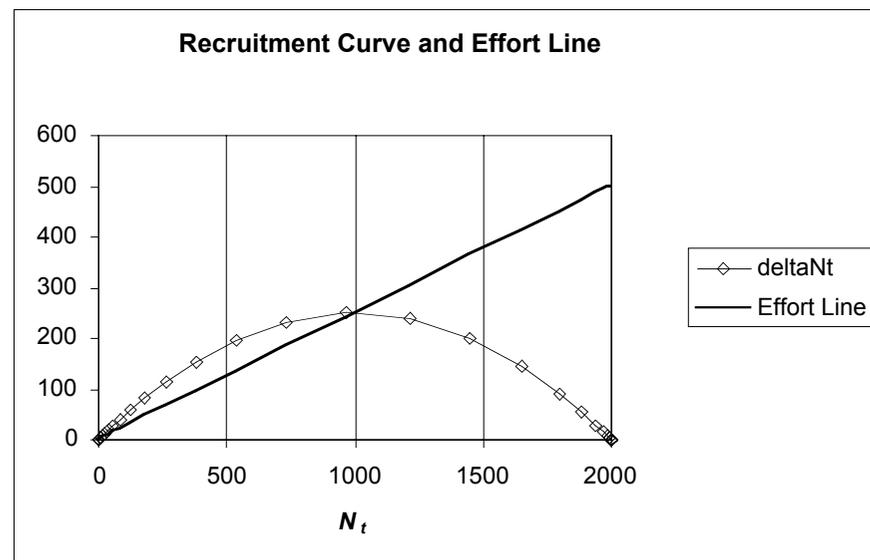


Figure 5 Note that the effort line (straight line sloping upward to the right), crosses the recruitment curve at its peak. This observation will be important in interpreting later developments of the model.

Fixed-Quota versus Fixed-Effort: First Questions

1. What are the effects of different values of R and K on catches and population sizes in the two harvest models?
2. What are the effects of different values of Q on catches and population size in the population harvested under a fixed-quota scheme?
3. What are the effects of different values of E on catches and population size in the population harvested under a fixed-quota scheme?
4. How does the maximum sustainable yield (MSY) relate to R and/or K ?
5. Is there any difference between fixed-quota and fixed-effort harvesting in terms of risk of driving the harvested population to extinction?

Fixed-Quota versus Fixed-Effort: Analysis

Now that you have tried out different population parameters and harvest quotas or efforts, let's do a little analysis. We will figure out the largest number of individuals we can harvest from the population indefinitely—the maximum sustainable yield. Then we'll plug that into our harvest models and see how they respond to errors in our estimates of population growth rate or carrying capacity or natural variations in these population parameters.

Remember that $\Delta N_t = N_{t+1} - N_t$, or by rearranging terms, $N_{t+1} = N_t + \Delta N_t$. So, if we harvest ΔN_t we get $N_{t+1} = N_t + \Delta N_t - \Delta N_t$, or simply $N_{t+1} = N_t$. In other words, we can harvest ΔN_t without changing the population size. This is a sustainable harvest, because recruitment (ΔN_t) depends on population size, and if population size remains unchanged, recruitment will also remain unchanged, and we can harvest the same number again next year.

The next step is to figure out what the maximum recruitment is, because that would be our maximum sustainable yield - the largest harvest we could take indefinitely.

Restore any changed model parameters to the values given at the beginning of the Procedures. Examine your graph of recruitment (ΔN_t) versus population size (N_t) for the unharvested population. At what value of N_t is recruitment (ΔN_t) greatest? How is this value of N_t related to K ?

You should be able to see that ΔN_t is maximum at $N_t = K/2$. To prove that this is true would require using the calculus on the continuous-time logistic model, and we'll spare you that. But you should be able to see by looking at your graphs that this is at least approximately true for the discrete-time model.

Now let's use a little algebra to figure out what recruitment is when it's at its maximum. We begin with the logistic growth equation (Equation 1, presented earlier):

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right)$$

If we subtract N_t from both sides we get

$$N_{t+1} - N_t = RN_t \left(\frac{K - N_t}{K} \right)$$

Remember that $N_{t+1} - N_t = \Delta N_t$, so

$$\Delta N_t = RN_t \left(\frac{K - N_t}{K} \right)$$

Earlier we figured out that ΔN_t is maximum when $N_t = K/2$, so let's substitute $K/2$ for N_t and see what we get for recruitment.

$$\Delta N_t = R \left(\frac{K}{2} \right) \left(\frac{K - K/2}{K} \right)$$

$$\Delta N_t = \left(\frac{RK}{2} \right) \left(\frac{K}{K} - \frac{K(1/2)}{K} \right)$$

The K 's in the right-hand parentheses cancel out, and we're left with

$$\Delta N_t = \left(\frac{RK}{2} \right) \left(1 - \frac{1}{2} \right)$$

$$\text{which is } \Delta N_t = \left(\frac{RK}{2} \right) \left(\frac{1}{2} \right) = \frac{RK}{4}$$

This is the maximum sustainable yield. Note that, as we said earlier, MSY is related to both R and K . Applying this value of the MSY to the fixed-quota harvest model is easy: simply set the quota $Q = RK/4$. The initial values given in the Procedures fit these requirements. If you have changed parameter values, put them back to the original values given in the Instructions.

Examine your graph of recruitment and fixed quota versus population size. You should see that the quota line just touches the recruitment curve at its maximum. At that point, the harvest exactly equals the recruitment, and the point of contact between the two curves represents an equilibrium - the maximum sustainable yield. Later, you will explore what happens if the population starts above or below this equilibrium. For now, ask yourself whether this equilibrium is stable or unstable. The answer may surprise you.

Applying our knowledge of the maximum sustainable yield to the fixed-effort model is almost as easy. We need to figure out a value of E that will give us a harvest equal to $RK/4$ when $N_t = K/2$, or $EN_t = RK/4$ when $N_t = K/2$.

If we substitute $K/2$ for N_t , we get

$$E \frac{K}{2} = \frac{RK}{4}$$

Multiplying both sides by 2, and dividing both sides by K , gives us

$$E = \frac{R}{2}$$

Another way to get the same answer, which may be more intuitive, follows. Look again at your graph of recruitment versus population size with the effort line superimposed. We want to calculate the slope (E) of the effort line. Because the effort line is a straight line, we can calculate its slope if we know the coordinates of any two points along it:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

You can see that the effort line passes through the origin, so the coordinates of one point are $(0,0)$. The line also passes through the maximum recruitment point, whose coordinates are $(K/2, RK/4)$. If you don't see where the latter coordinates came from, look back over the derivation above for maximum sustainable yield.

Plugging these coordinates into the formula for slope (E), we get

$$E = \frac{y_2 - y_1}{x_2 - x_1} = \frac{RK/4 - 0}{K/2 - 0} = \frac{RK}{4} \times \frac{2}{K} = \frac{R}{2}$$

This completes our algebraic analysis. To summarize, we found that:

- Maximum sustainable yield (MSY) is $RK/4$.
- We can harvest the MSY by setting a fixed quota to $RK/4$.
- We can also harvest the MSY by setting a fixed effort to $R/2$.

Armed with this analysis, you should now try to answer questions 6–16. Again, the answers are explained later, but you should attempt them on your own first.

Fixed-Quota versus Fixed-Effort: More Questions

6. What happens to a population harvested at the MSY under a fixed quota if it starts out at its carrying capacity?
7. What happens to a population harvested at the MSY under a fixed quota, if it starts out at less than half its carrying capacity?
8. What happens to a population harvested at the MSY under a fixed-effort strategy, if it starts out at its carrying capacity?
9. What happens to a population harvested at the MSY under a fixed-effort strategy, if it starts out at less than half its carrying capacity?
10. You can calculate exact values for the fixed quota and the fixed effort that produce the maximum sustainable yield because you *know* the exact values of R and K for the harvested populations. In the real world, however, managers must *estimate* these values, and estimates are always subject to error. What happens to each of the harvested populations if you overestimate K ? In other words, what happens if K is actually smaller than you think it is?
11. What happens if you overestimate R ?
12. What happens if you set the quota or the effort too high?
13. Do you see another reason why setting a fixed quota at the maximum sustainable yield is a risky strategy?
14. Is the fixed-effort harvest less risky?
15. What happens if managers decide to build a safety margin into a fixed-quota harvest?
16. In answering questions 6–15, you will have seen two ways in which a fixed-effort harvest is less risky than a fixed-quota harvest. A fixed-effort harvest is not completely without risk, however. What happens to a population harvested under a fixed-effort strategy if the effort is set too high? How does a high effort increase the risk of extinction in a fixed-effort harvest?

Adding Annual Variation

So far, we have assumed that R and K are constants, but reality is rarely so simple. We can improve our models by varying R or K , but before we do so, let's consider what effects such variation would produce. If K varies from year to year, the impact on the population is greatest when N is close to K . If R varies from year to year, the effect on population growth is greatest when N is in the range of roughly $\frac{1}{4}K$ to $\frac{3}{4}K$. You can confirm these statements for yourself, using the formulae suggested below, if you wish.

Because we are emphasizing a scenario in which harvesting at the MSY keeps the population at about $\frac{1}{2}K$, we will explore the consequences of variation in R . Accordingly, we modify our basic logistic equation by adding a random term to R :

$$N_{t+1} = N_t + (R + SDR)N_t \left(\frac{K - N_t}{K} \right)$$

In this equation, D is a number that varies at random from -1 to 1 . The parameter S is a scaling factor that determines the size of this random variation relative to the value of R . For example, if you set R to 0.5 and S to 0.5 , this equation will vary the value of R from 0.25 to 0.75 .

If you want to vary K , you can use a similar modification. To simplify matters, we begin by rewriting the logistic equation slightly:

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right)$$

$$N_{t+1} = N_t + RN_t \left(\frac{K}{K} - \frac{N_t}{K} \right)$$

$$N_{t+1} = N_t + RN_t \left(1 - \frac{N_t}{K} \right)$$

Adding stochastic variation to K gives us

$$N_{t+1} = N_t + RN_t \left(1 - \frac{N_t}{(K + SDK)} \right)$$

This kind of random error is called **stochasticity**, and the models you are about to build are called **stochastic** models. Your previous models were **deterministic** (i.e., without stochasticity).

INSTRUCTIONS

D. Harvest models with annual variation.

1. Modify your spreadsheet by setting up a place for a scaling factor. Enter an initial value of 0.5.

2. Modify your formulae for population size of the fixed-quota population to include stochastic variation.

3. Copy the modified formula down the column.

4. Change the formula for the fixed-effort harvest similarly.

ANNOTATION

In cell C6, enter the label "S =". In cell D6, enter the value 0.5.

If you prefer, you can copy your unmodified spreadsheet into a fresh worksheet, or save a copy under a different name. This will allow you to complete this part of the exercise, and retain the deterministic models you have built so far.

The *old* formula should read `=IF(D12+B6*D12*(B$7-D12)/B$7-F9>0,D12+B6*D12*(B$7-D12)/B$7-F9,0)`.

The *new* formula should read `=IF(D12+(B6+D6*(RAND()*2-1)*B$6)*D12*(B$7-D12)/B$7-$F$9>0,D12+($B$6+$D$6*(RAND()*2-1)*B$6)*D12*(B$7-D12)/B$7-F9,0)`.

This reflects a change in the mathematical formula for the fixed-quota population from

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - Q$$

to

$$N_{t+1} = N_t + (R + SDR)N_t \left(\frac{K - N_t}{K} \right) - Q$$

We added random numbers to the spreadsheet formula to simulate stochasticity. The spreadsheet has a built-in random number generating function called **RAND()**. The portion of the formula `=RAND()*2-1` generates a random number between -1 and 1.

The *old* formula should read `=IF(G12+B6*G12*(B$7-G12)/B$7-I12>0,G12+B6*G12*(B$7-G12)/B$7-I12,0)`.

The *new* formula should read

=IF(G12+(\$B\$6+\$D\$6*(RAND()*2-1)*\$B\$6)*G12*(\$B\$7-G12)/\$B\$7-I12>0,G12+(\$B\$6+\$D\$6*(RAND()*2-1)*\$B\$6)*G12*(\$B\$7-G12)/\$B\$7-I12,0).

The change in the mathematical formula for the fixed-effort harvest population is from

$$N_{t+1} = N_t + RN_t \left(\frac{K - N_t}{K} \right) - EN_t$$

to

$$N_{t+1} = N_t + (R + SDR)N_t \left(\frac{K - N_t}{K} \right) - EN_t$$

5. Copy the modified formula down the column.

6. Return all the model parameters to their original values, if necessary.

7. You do not need to change any of your graphs, or make any new ones. Save your work!

$R = 0.5$
 $K = 2000$
 $Q = 250$
 $E = 0.25$

Annual Variation: Questions

17. Does the addition of stochastic variation to the model increase the risk of extinction for a population harvested at the MSY under a fixed quota?
18. Does the addition of stochastic variation to the model increase the risk of extinction for a population harvested at the MSY by fixed effort?
19. Does reducing a fixed quota below the MSY (i.e., building in a safety margin) prevent extinction of a stochastically varying population?
20. Does the addition of stochastic variation to the model increase the risk of extinction for a population harvested above the optimal effort (i.e., the effort that yields the MSY)?

Bonus Analysis (Advanced)

So far, the fixed-effort harvest looks pretty safe. But what if the population shows the Allee effect (zero recruitment at small population sizes)? Try modifying your logistic equation so that per capita recruitment becomes zero below some positive threshold value of N_t and see if that changes your assessment of the fixed-effort harvesting strategy.