

# 1

## MATHEMATICAL FUNCTIONS AND GRAPHS

### Objectives

- Learn how to enter formulae and create and edit graphs.
- Familiarize yourself with three classes of functions: linear, exponential, and power.
- Explore effects of logarithmic plots on graphs of each kind of function.

### INTRODUCTION

This exercise serves two main purposes: to allow you to practice some of the procedures outlined in “Spreadsheet Hints and Tips,” and to acquaint you with three classes of mathematical functions. Biology, like all sciences, uses mathematical relations to describe natural phenomena. In many cases, the mathematics is only implied, as in any graph of one variable against another. In other cases, it is made explicit in the form of an equation. Such relationships take a variety of forms, but you will encounter three classes of relationships with some regularity in textbooks and journal articles: linear functions, exponential functions, and power functions.

For example, the number of lizard species in a given area of desert habitat rises *linearly* with the length of the growing season; a bacterial population introduced into an empty vial of nutrient broth will grow *exponentially* (at least for a time); and the number of species on an island is a *power* function of the island’s area.

A mathematical **function** relates one **variable** to another. For example, we may say that the death rate in a population is a function of population density, meaning that death rate and population density (both numbers that change from population to population, and even within a population—i.e., numbers that are “variable”) are related in some way. By writing an equation, we can specify precisely how these variables relate to one another.

For convenience, we usually refer to one variable as the **independent variable** and the other as the **dependent variable**, and we speak of the dependent variable “depending on” the independent variable. For example, we may say that death rate depends on population density. If one variable is clearly a cause of the other, we take the cause as the independent variable and the effect as the dependent variable. But in many cases, cause and effect relationships are not clear, or each variable may in a sense cause the other and be an effect of the other. Population

density and death rate offer an example of such a mutual cause-effect relationship. In such cases, our choice of which variable to treat as independent and which to treat as dependent is a matter of convenience or convention.

As a matter of convention, we denote the independent variable as  $x$  and plot it on the horizontal axis of a graph, and we denote the dependent variable as  $y$  and plot it on the vertical axis.

More strictly speaking, a function is a rule that produces one and only one value of  $y$  for any given value of  $x$ . Some equations, such as  $y = \sqrt{x}$ , are not functions because they produce more than one value of  $y$  for a given value of  $x$ . We can often treat such equations as functions by imposing some additional rule; in this case, we might restrict ourselves to positive square roots.

Functions take a variety of forms, but to begin with, we will concern ourselves with the three broad categories of functions mentioned earlier: linear, exponential, and power. **Linear functions** take the form

$$y = a + bx$$

where  $a$  is called the  **$y$ -intercept** and  $b$  is called the **slope**. The reasons for these terms will become clear in the course of this exercise. **Exponential functions** take the form

$$y = a + q^x$$

**Power functions** take the form

$$y = a + kx^p$$

Note the difference between exponential functions and power functions. Exponential functions have a constant base ( $q$ ) raised to a variable power ( $x$ ); power functions have a variable base ( $x$ ) raised to a constant power ( $p$ ). The base is multiplied by a constant ( $k$ ) after raising it to the power ( $p$ ).

## PROCEDURES

The left-hand column of instructions gives rather generic directions; the right-hand column gives a step-by-step breakdown of these and explanatory comments or annotations. If you are not familiar with an operation called for in these instructions, refer to "Spreadsheet Hints and Tips."

Try to think through and carry out the instructions in the left-hand column before referring to the right-hand column for confirmation. This way, you will learn more about using the spreadsheet, rather than simply following directions. We hope that, with practice, you will gain enough skill in using the spreadsheet that you will be able to modify our models, or create your own from scratch, to suit your own uses.

Your goals in this exercise are to learn how to use a spreadsheet program to calculate and graph these functions and to see how these graphs look with linear and logarithmic axes. In achieving these goals, you will learn about the behavior of these classes of functions, how to use formulae, how to make graphs, and the utility of logarithmic plots. Save your work frequently to disk!

**INSTRUCTIONS**

**A. Set up the spreadsheet.**

1. Enter titles and headings through Row 9, as shown in Figure 1. You need not enter the text shown in Rows 2 through 6, but if you don't enter the text, leave these rows blank so that the cell addresses in your formulae will match the ones given in these instructions.

**Linear Functions**

2. Set up a linear series from 0 to 9 in cells A10–A19. This will provide values for the independent variable  $x$ .

3. In cell B10, enter a spreadsheet formula that expresses the equation shown in cell B9.

4. Copy the formula in cell B10 down the column through cell B19.

5. Enter formulae for the equations shown in cells C10, D10, and E10 into cells C11, D11, and E11, respectively.

6. Copy these formulae down their respective columns.

**ANNOTATION**

These are all literals, so select each cell by clicking in it with the mouse, then type in each title or heading. Use the delete (backspace) key or highlight and overwrite to correct errors.

	A	B	C	D	E	F
1	Functions and Graphs					
2	The first part of this exercise will familiarize you with several kinds of mathematical					
3	functions, entering formulae, and graphing in Excel.					
4	The second part will compare functions.					
5						
6	Part 1: Kinds of Functions					
7	Independent					
8	variable					
		Linear functions				
9	(x)	$y=5+1x$	$y=0+5x$	$y=10+5x$	$y=60-5x$	
10	0	5	0	10	60	
11	1	6	5	15	55	
12	2	7	10	20	50	
13	3	8	15	25	45	

Figure 1

Enter the value 0 as a literal in cell A10.  
In cell A11, enter the formula  $=A10+1$ . Copy the formula in cell A11.  
Select cells A12–A19. Paste.

In cell B10, type the formula  $=5+1*A10$ .  
We could omit the 1 in the equation and in the formula, but we keep it for consistency with the others.

Copy the contents of cell B10.  
Select cells B11–B19. Paste.

These should be:  
Cell C10:  $=0+5*A10$   
Cell D10:  $=10+5*A10$   
Cell E10:  $=60-5*A10$

Select cells C10–E10. Copy.  
Select cells C11–E19. Paste.

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7. Adjust the widths of columns to accommodate text and numbers.

**Exponential Functions**

8. Enter titles and headings in Rows 21 and 22.

9. Set up a linear series from 0 to 9 in cells A23–A32. This will provide values for the independent variable  $x$ .

10. In cells B23–D23, enter spreadsheet formulae that express the equations shown in cells B22–D22.

11. Copy the formulae in cells B23–D23 into cells B24–D32.

12. If needed, adjust column widths to accommodate text and numbers.

**Power Functions**

13. Enter titles and headings in Rows 34 and 35.

Select the column(s) to be modified.

You can either open Format | Column | AutoFit Selection, or click and drag column boundaries at the top of the page to achieve the desired widths.

These are all literals, so enter them as before (see Step 1).

	A	B	C	D	E
21		Exponential functions			
22	x	$y=0+1.1^x$	$y=0+1.5^x$	$y=0+1.5^{-x}$	
23	0	1.00	1.00	1.00	
24	1	1.10	1.50	0.67	
25	2	1.21	2.25	0.44	

Figure 2

Enter the number 0 as a literal in cell A23.

In cell A24, enter the formula =A23+1.

Copy the formula in cell A24.

Select cells A25–A32. Paste.

These should be

Cell B23: =0+1.1^A23

Cell C23: =0+1.5^A23

Cell D23: =0+1.5^-A23

We could omit the zeros in the equations and in the formulae, but we keep them for consistency with the others.

Select cells B23–D23. Copy.

Select cells B24–D32. Paste.

At this point, your spreadsheet should contain the values shown above.

See Step 7.

These are all literals, so enter them as before (see Step 1).

	A	B	C	D	E
34		Power functions			
35	x	$y=0+x^2$	$y=0+x^{0.5}$	$y=0+x^{-0.5}$	
36	1	1.00	1.00	1.00	
37	2	4.00	1.41	0.71	
38	3	9.00	1.73	0.58	

Figure 3

14. Set up a linear series from 1 to 10 in cells A36–A45. This will provide values for the independent variable  $x$ .

15. In cells B36–D36, enter spreadsheet formulae that express the equations shown in cells B35–D35.

16. Copy the formulae in cells B36–D36 into cells B37–D45.

**Comparing Functions**

17. Enter titles and headings in Rows 47–55. Also enter the values shown for the parameters (constants).

18. Set up a linear series from 1 to 10 in cells A56–A65.

19. Enter formulae into cells B56–D56 to calculate the functions in cells B55–D55.

20. Copy the formulae down their columns.

**Your spreadsheet is complete. Save your work!**

Enter the number 1 as a literal in cell A36.  
In cell A37, enter the formula  $=A36+1$ .

Copy the formula in cell A37.  
Select cells A38–A45. Paste.

Note that this differs from previous examples by starting at 1 rather than 0. We will explain why later.

These should be

- Cell B36:  $=0+A36^2$
- Cell C36:  $=0+A36^{0.5}$
- Cell D36:  $=0+A36^{-0.5}$

Again, we could omit the zeros in the equations and in the formulae, but we keep them for consistency with the others.

Select cells B36–D36. Copy.  
Select cells B37–D45. Paste.

At this point, your spreadsheet should contain the values shown above.

These are all literals, so enter them as before (see Step 1).

	A	B	C	D	E
47	Part 2:Comparing Functions				
48		Parameters (constants)			
49		y-Intercept (a)	0		
50		Slope (b)	1		
51		Base (q)	2		
52		Power (p)	3		
53					
54		Linear	Exponential	Power	
55	x	$y=a+bx$	$y=a+q^x$	$y=a+x^p$	
56	1	1	2	1	
57	2	2	4	8	
58	3	3	8	27	

**Figure 4**

Enter the number 1 as a literal in cell A56.  
In cell A57, enter the formula  $=A56+1$ .

Copy the formula in cell A57.  
Select cells A58–A65. Paste.

The formulae should read:

- Cell B56:  $=\$C\$49+\$C\$50*A56$
- Cell C56:  $=\$C\$49+\$C\$51^A56$
- Cell D56:  $=\$C\$49+A56^{\$C\$52}$

Select cells B56–D56. Copy  
Select cells B57–D65. Paste.

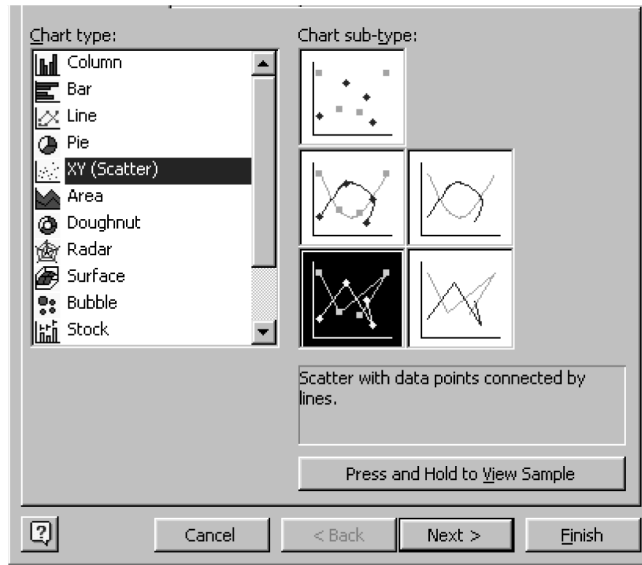
At this point, your spreadsheet should contain the values shown in Figure 4.

**B. Create graphs.**

**Linear Functions**

1. Graph all four linear functions on the same graph.

Select the contiguous block of cells from cell A9 through cell E19. Note that you should select the column headings as well as the data to be graphed. This lets the program label the graph legend correctly. Click on the Chart Wizard icon or open Insert | Chart. In the Chart Type dialog box, select XY (Scatter). Then, from the chart subtypes shown, choose the one at bottom left, which has data points connected with straight lines.

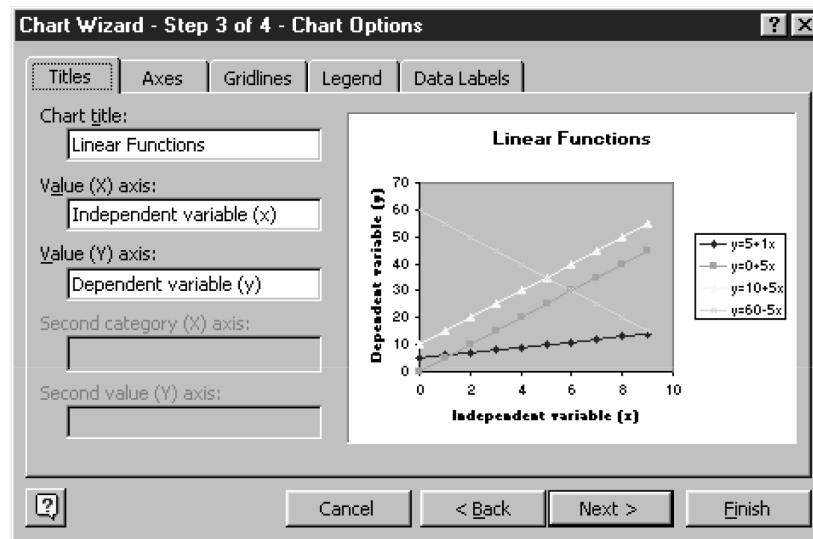


**Figure 5**

Click the Next button.

In the Chart Source Data dialog box, select Series in Columns. This will probably already be selected for you, in which case you need only click on the Next button.

In the Chart Options dialog box, enter a figure title and axis labels as shown in Figure 6.



**Figure 6**

2. Edit your graph to improve readability. Change to an uncolored background.

Note the tabs across the top of the dialog box. Clicking on one of these will take you to another page of chart options. We usually go to the gridlines page and remove the horizontal gridlines that appear by default because we find them distracting. This has already been done in Figure 6. Click the Next button.

In the Chart Location dialog box, select Place Chart: As Object In: Sheet 1 and click on the Finish button.

Often, the shaded background and default colors of data markers and lines are difficult to see and print poorly, especially on black-and-white printers. To change to an uncolored (clear) background, double-click inside the graph axes, away from any lines or data markers, and you should see the dialog box shown below. Click on the buttons labeled None for Border and Area, as shown in Figure 7.

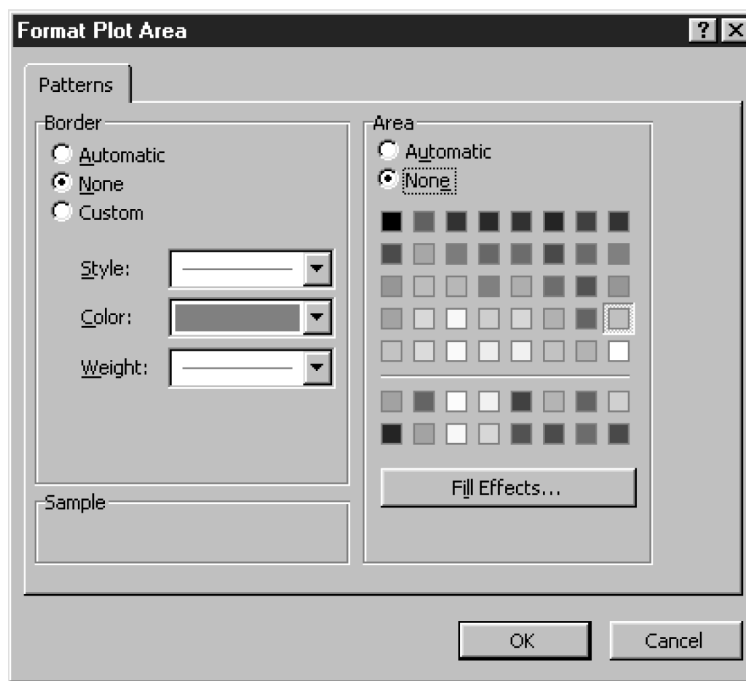


Figure 7

3. Make all data lines and markers black and give each function an easily distinguished marker or line type.

Double-click on a data point marker, and you should see the dialog box in Figure 8.

The left-hand section offers several options for formatting the line connecting data points. Click and hold on the arrow in the box labeled Color and a color palette will pop up. Still holding down the mouse button, select Black. You can change the style of the line (solid, dashed, dotted, etc.) and its weight (thickness) similarly. In general, you should *not* use the smoothed line option.

The right-hand section offers options for formatting data markers. Change the foreground and background colors to black as you did for line color. You can use the Style pop-up menu to choose the shape of the data marker. To make hollow markers, choose No Color from the color palette for background color.

Edit each data series similarly, making all black and choosing easily distinguished markers or line-styles.

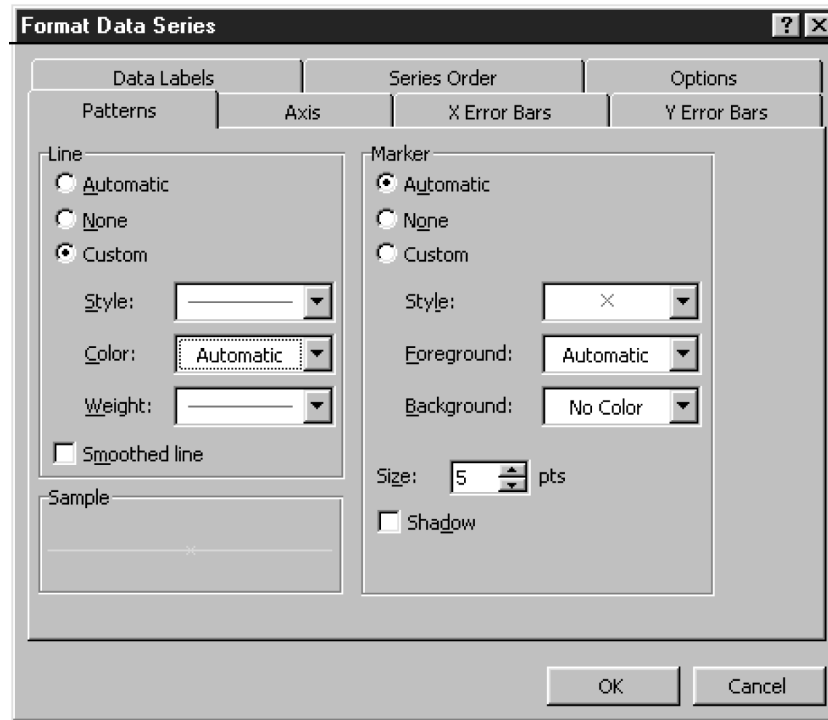


Figure 8

4. Your graph should now resemble the one in Figure 9.

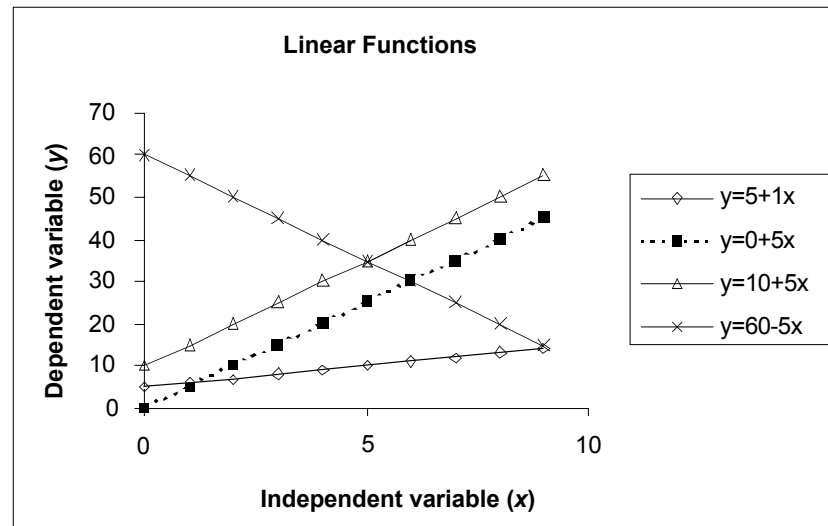


Figure 9

5. If the graph obscures cells A19–E9 of your spreadsheet, drag it to the right so that those cells are visible.

**Exponential Functions**

6. Graph all three exponential functions on a new graph.

Click once inside the box around the graph, but outside the graph axes. The graph box should now have small, square “handles” at the middle of each side. If it does not, try clicking in a different place inside the graph box. Press and hold the mouse button while dragging the graph to the desired location. If only part of the graph moves, rather than the entire graph moving as a unit, open Edit | Undo Move and try again.

Select the contiguous block of cells A22–D32. Note that you should select the column headings as well as the data to be graphed. This lets the program label the graph legend correctly.



7. Edit your graph to improve readability.

Click on the Chart Wizard icon or open Insert | Chart.  
Follow the steps for graphing linear functions given in Section B1.

Follow the steps given in Section B2 on linear functions: Remove gridlines and label the graph and its axes. Remove background color and change all lines and data markers to black. Choose markers and line types so that different functions are clearly labeled. When you are done, your graph should look something like the graph in Figure 10.

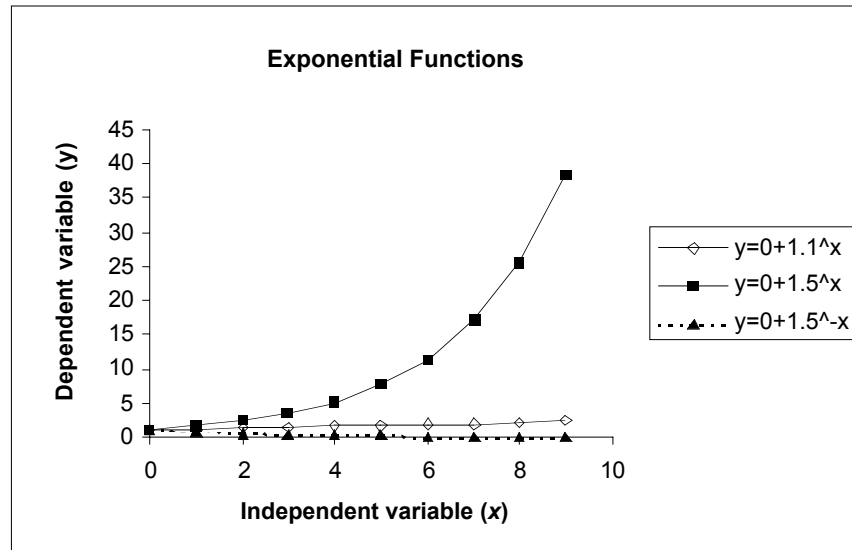


Figure 10

8. Change the vertical axis to a logarithmic scale.

Double-click on the vertical axis. A dialog box will appear. Click on the tab labeled Scale. The page shown in Figure 11 will appear. Click in the box labeled Logarithmic Scale. Do *not* click on the OK button yet.

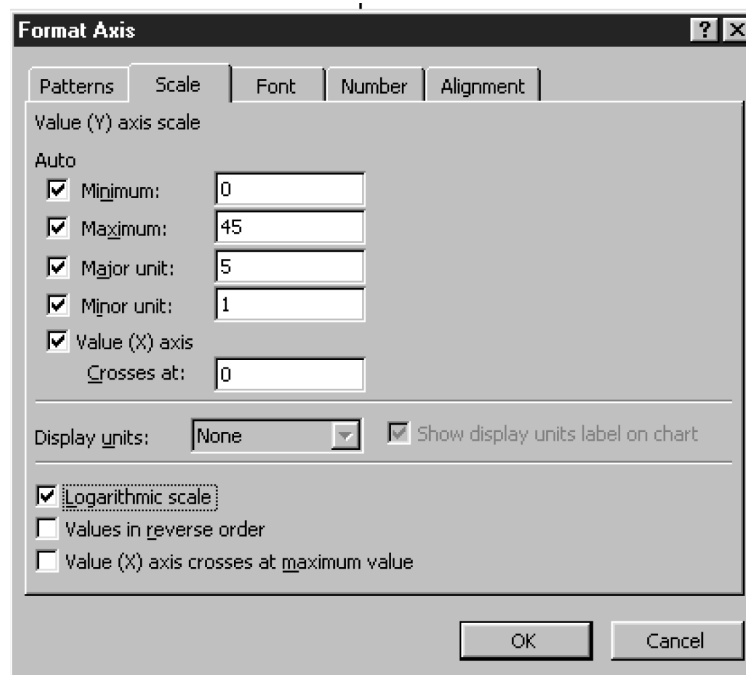


Figure 11

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9. Change the numbers on the vertical axis to display two decimal places.

Click on the tab labeled Number. The page shown in Figure 12 will appear. Select Number from the category list on the left. Use the little arrows next to the Decimal Places box to select 2 decimal places.

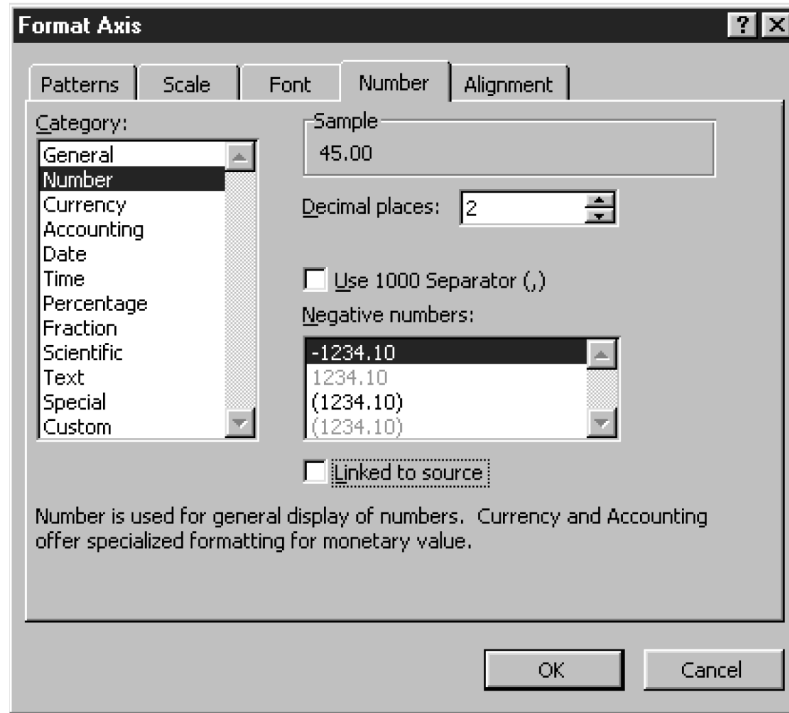


Figure 12

Now click on the OK button.

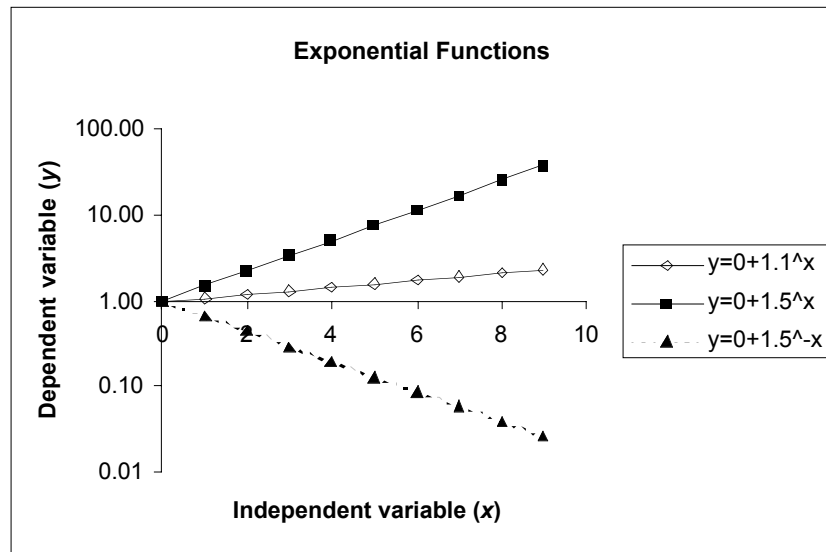


Figure 13

10. Your graph should now resemble the one in Figure 13.

Note that exponential functions are graphed as straight lines when the vertical axis is logarithmic and the horizontal axis is linear. A graph with such axes is called a **semi-log plot**. Plotting variables on a semi-log plot is a good way to test for an exponential relationship.

**Power Functions**

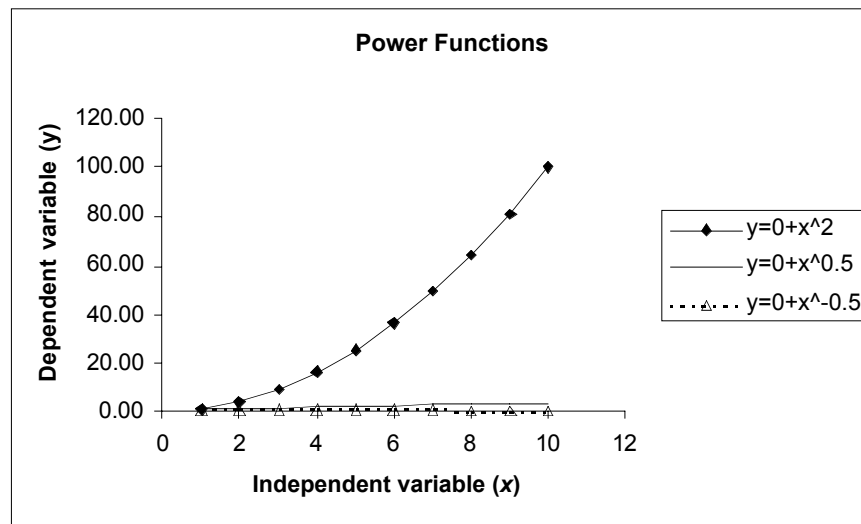
11. Graph all three power functions on a new graph.

12. Edit your graph to improve readability. Your graph should resemble the one in Figure 14. Graphing each function separately reveals the shapes of their graphs.

Select the contiguous block of cells A35–D45. Note that you should select the column headings as well as the data to be graphed. This lets the program label the graph legend correctly.

Click on the Chart Wizard icon or open Insert | Chart. Follow the steps given in Section B1 on linear functions.

Follow the steps given in Section B2 on linear functions: Remove gridlines and label the graph and its axes. Remove background color and change all lines and data markers to black. Choose markers and line types so that different functions are clearly labeled.



**Figure 14**

The graph of  $y = x^2$  resembles an exponential function but, as we will show shortly, it is not. The other functions lie almost on top of the  $x$ -axis.

13. Change the vertical axis to a logarithmic scale.

14. Change the numbers on the vertical axis to display one decimal place.

15. Change the horizontal axis to a logarithmic scale. Your graph should now resemble the one in Figure 15.

Double-click on the vertical axis.

In the dialog box, click on the Scale tab and select Logarithmic Scale. Do not click OK yet.

Click the Number tab, and use the Decimal Places box to select 1 decimal place. Now click OK.

Note that none of the functions appears as a straight line; this tells you that they are not exponential functions.

Follow the same procedure that you used in changing the vertical axis to a logarithmic scale.

Note that all these power functions are graphed as straight lines when both axes are logarithmic. A graph with such axes is called a **log-log plot**. Plotting variables on a log-log plot is a good way to test for a power relationship.

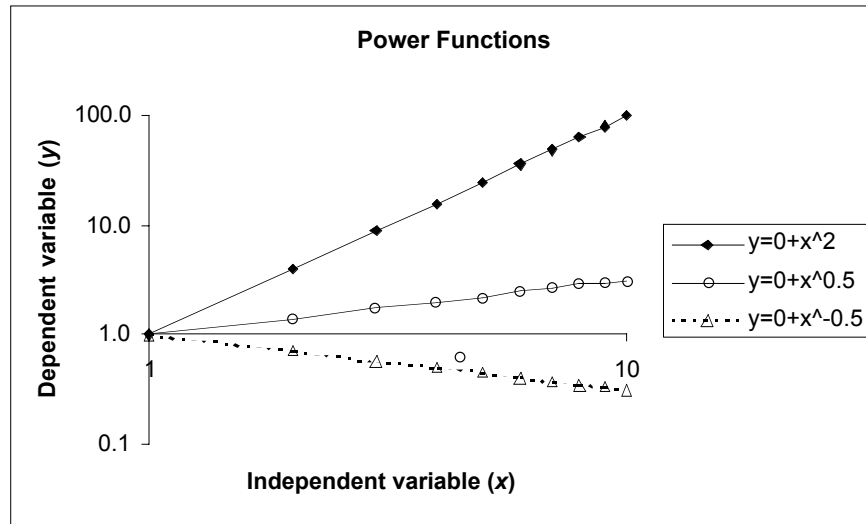


Figure 15

**Comparing Functions**

16. Graph the three functions in cells A55–D65 on a new graph.

17. Edit your graph to improve readability. Your graph should resemble the one in Figure 16.

Select cells A55–D65. Note that you should select the column headings as well as the data to be graphed. This lets the program label the graph legend correctly. Click on the Chart Wizard icon or open Insert | Chart. Follow the steps given in the section on linear functions.

Follow the steps given in Section B2 on linear functions: Remove gridlines and label the graph and its axes. Remove background color and change all lines and data markers to black. Choose markers and line types so that different functions are clearly labeled.

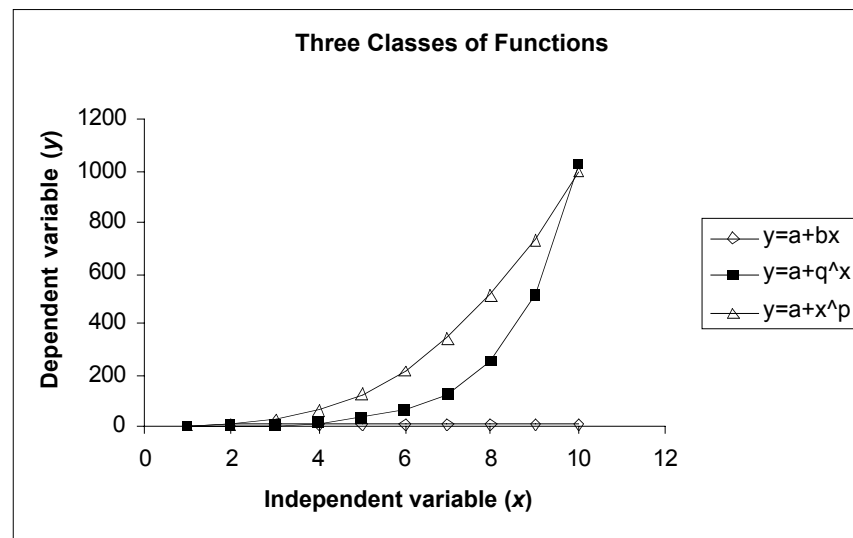


Figure 16

18. Experiment with different combinations of logarithmic and linear axes.

19. Experiment with different values of  $y$ -intercept, slope, base, and power, and observe the effects on the graph.

Try:

- Both axes linear
- Logarithmic  $x$ -axis, linear  $y$ -axis(semi-log)
- Both axes logarithmic (log-log)

See instructions above for details of changing axis scaling.

Simply enter new values in the cells labeled “Parameters” (constants)—cells C49 through C53. You do not need to edit the formulae.

## QUESTIONS

1. How does changing the value of the  $y$ -intercept ( $a$ ) affect each of the kinds of functions? Enter different values in cell C49 and observe the effects on your graph of three kinds of functions. The effects may be difficult to see at first, because the spreadsheet automatically rescales the  $y$ -axis to accommodate values to be graphed. Be sure to note the values along the  $y$ -axis in your comparisons. Also compare the four linear functions you graphed in step B1.
2. How does changing the value of the slope ( $b$ ) in cell C50 affect the linear function? Try values greater and less than zero. Also compare the four linear functions you graphed in step B1.
3. How does the exponential function look if you enter different values for the base ( $q$ ) in cell C51? Try values greater than one, equal to one, less than one, and less than zero. You will have to reformat the axes of your graph to see some of these effects. Also compare the three exponential functions you graphed in step B6.
4. How does the power function look if you enter different values for the power ( $p$ ) in cell C52? Try values greater than one, equal to one, less than one, and less than zero. You will have to reformat the axes of your graph to see some of these effects. Also compare the three power functions you graphed in step B11.
5. Find examples of all three kinds of functions in your textbook or in other books or papers about ecology or biology. Look for explicit equations and for graphs that imply these functions by their axis formats (both axes linear,  $y$ -axis logarithmic, or both axes logarithmic).

